Performance Assessment Task

Garden Design
Grade 3

This task challenges a student to use understanding of area and count squares to find the area of shapes on a grid. A student must be able to compare the area of shapes on a grid. A student must demonstrate knowledge of attributes and area in order to construct a different shape while maintaining an equal area.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with size length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

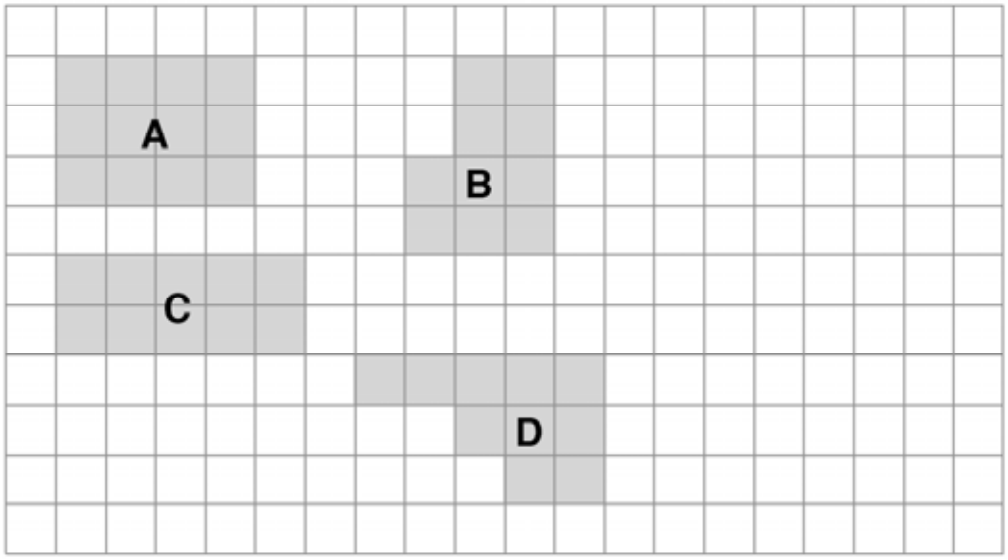
the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
3	2006	7	3	66%

Garden Design

This problem gives you the chance to:
- compare areas of shapes on a grid
- draw a shape with given area

Here is a plan of Martin's garden.
The shaded areas show where he plants flowers.

Scale: ☐ = 1 square unit



1. What is the area of shape A? _____ square units

2. Which shape has the largest area? _____
   Explain how you figured it out.

   _____

   _____

   _____

   _____

3. On the diagram above, draw a different shape that has the same area as shape A.
   Label your shape **E**.

7

Garden Design	Rubric	
The core elements of performance required by this task are: • compare areas of shapes on a grid • draw a shape with given area Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **12** square units	1	1
2. Gives correct answer: **A** Explains that the area of B = 10, the area of C = 10, the area of D = 10, so the area of A is the largest.	1 3x1	4
3. Draws a different shape with area 12 square units, and labels it E. *Partial credit* Draws a rotation of shape A.	2 (1)	2
Total Points		**7**

3rd Grade – Task 5: Garden Design

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess?

Do you think your students understand the idea of area? Look at their work in part 1. How many of your students put:

12	14	18	Shape name like square or rectangle	Other

How might a student come up with the answer 14? With 18?

What does a student need to understand in order to make a comparison?

What would you like to see in a justification?

How many of your students:

Gave values for all shapes	"I counted"	Picked Shape D	Confused area and perimeter	Other

What kind of activities can help students to develop and understand the logic of making a comparison?

What kind of activities help students see a need for calculating each area, to understand that just looking at a shape or looking at height or width may not give the correct answer?

How often do your students get opportunities to design their own shapes?

How is the mathematical demand of the task different from just finding the area? What are the different things students have to think about to design a shape to meet certain criteria?

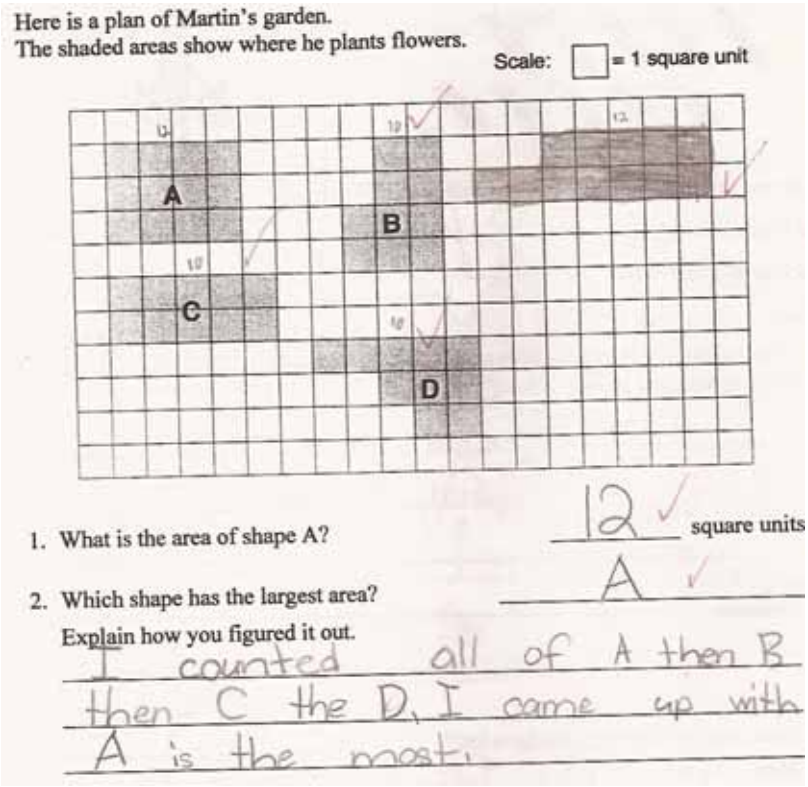
Look at their work on part 3. How many of your students drew:

Different Shape	Same Shape	Rotation of Shape A	Shape was too large	Shape was too small	Other

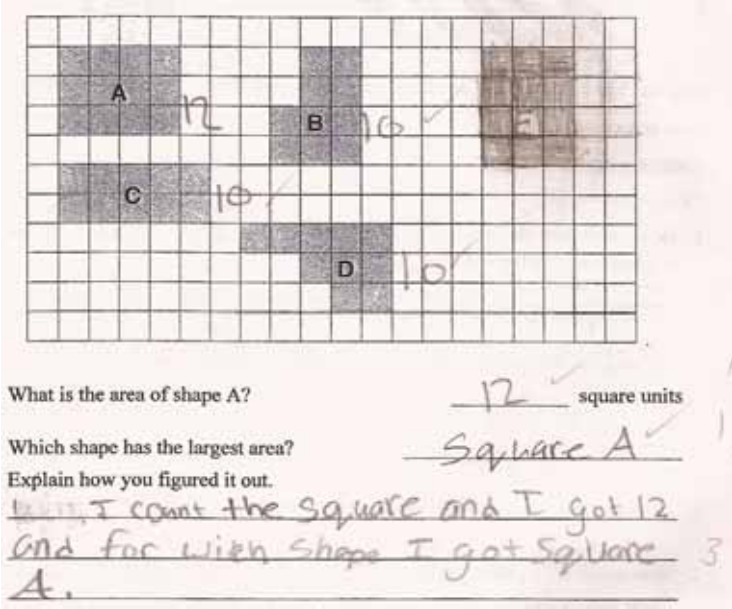
Looking at Student Work on Garden Design:

Student A is able to show the areas for all shapes. Therefore, to compare areas the student just needs to pick the largest value. Student A is also able to design a shape with the same area as shape A. Notice that the student labels the area for the new shape to check or justify the answer. Student B also makes a comparison, but draws a rotation of A instead of a different shape.

Student A

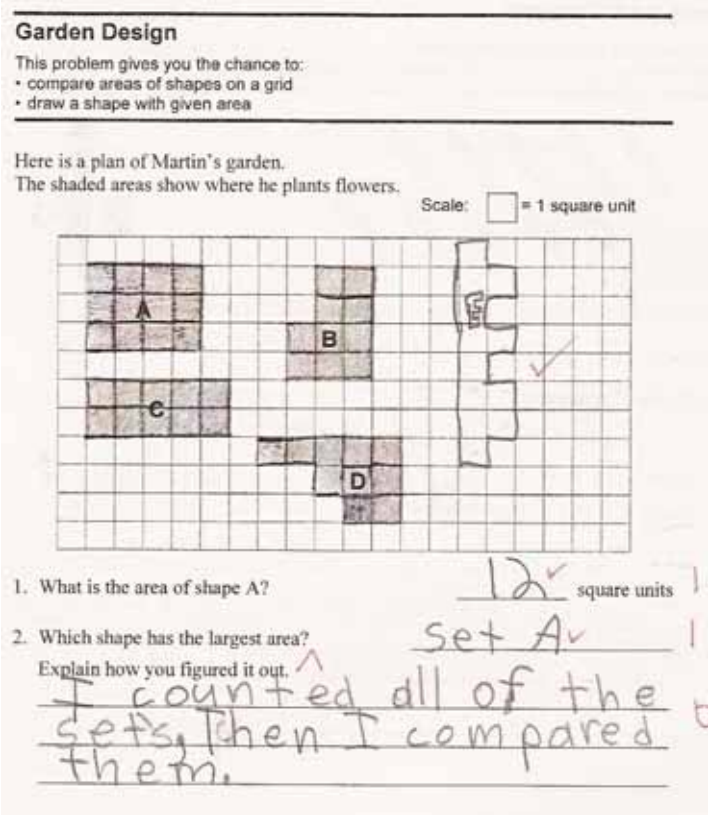


Student B



While Student C may have understood how to find the areas of all the shapes, the student does not quantify these areas. So the justification for the comparison is incomplete.

Student C



Garden Design

This problem gives you the chance to:
- compare areas of shapes on a grid
- draw a shape with given area

Here is a plan of Martin's garden.
The shaded areas show where he plants flowers.

Scale: ☐ = 1 square unit

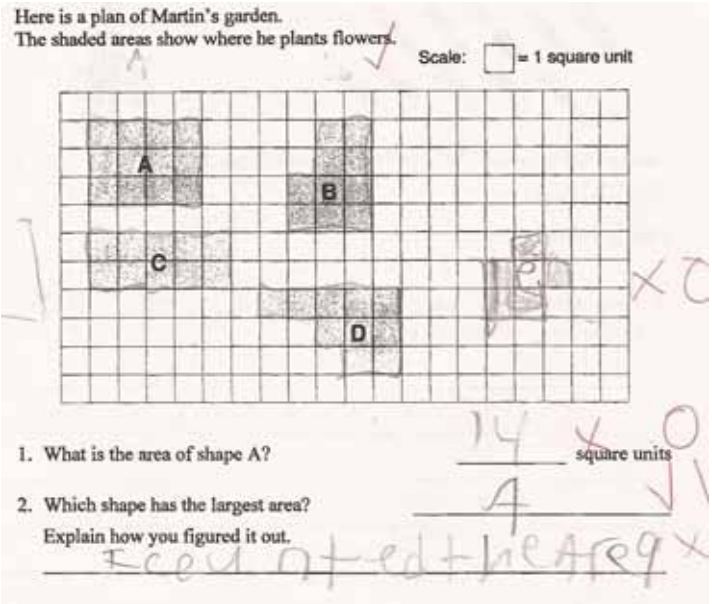
1. What is the area of shape A? 12 square units

2. Which shape has the largest area? Set A

Explain how you figured it out.
I counted all of the sets, then I compared them.

Student D confuses area and perimeter. Notice that shape E has neither the same area nor perimeter as shape A.

Student D



Here is a plan of Martin's garden.
The shaded areas show where he plants flowers.

Scale: ☐ = 1 square unit

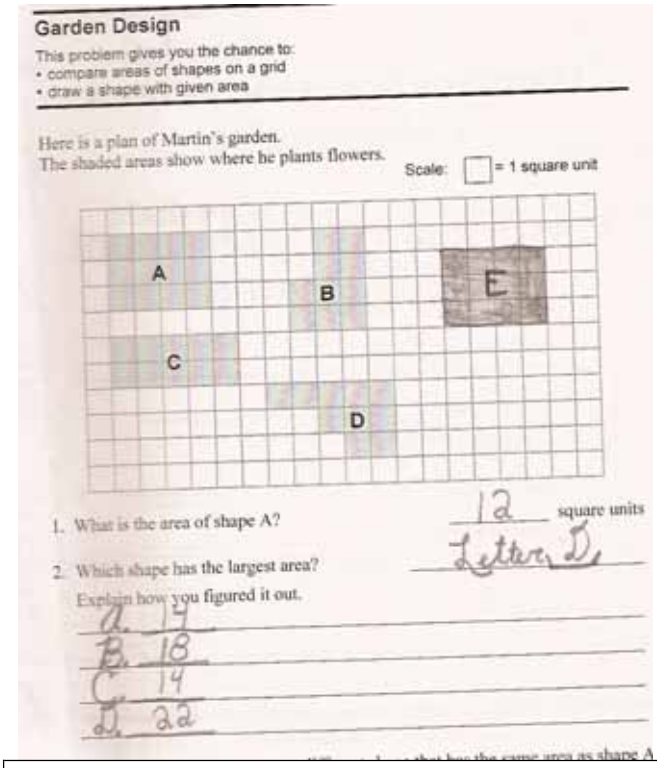
1. What is the area of shape A? 14 square units

2. Which shape has the largest area? A

Explain how you figured it out.
I counted the Area

Student E is inconsistent in her thinking. For part 1, the student finds the correct area for shape A. However in part 2 the student might be finding perimeter or something else. The student does not design a new shape with the same area as A, but just copies shape A.

Student E



Garden Design

This problem gives you the chance to:
- compare areas of shapes on a grid
- draw a shape with given area

Here is a plan of Martin's garden.
The shaded areas show where he plants flowers.

Scale: ☐ = 1 square unit

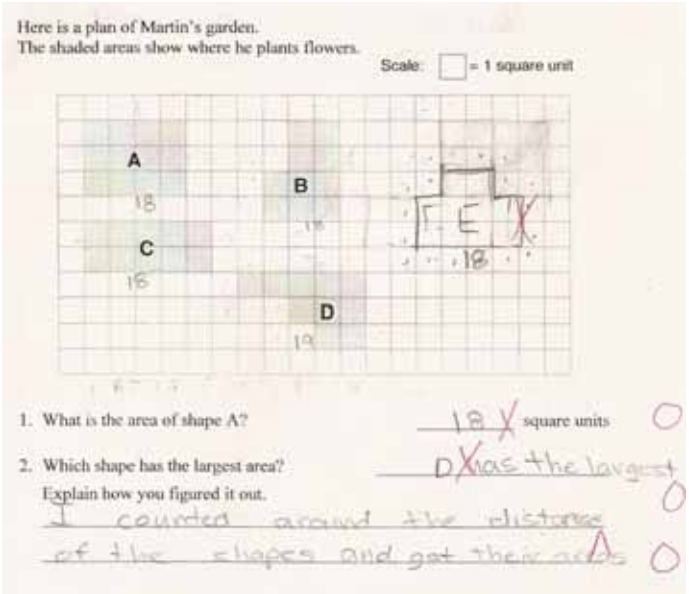
A B C D E

1. What is the area of shape A? 12 square units

2. Which shape has the largest area? Letter D.
Explain how you figured it out.
A. 12
B. 18
C. 14
D. 22

Student F counts the squares around the outside of the shape, which is even larger than the perimeter.

Student F



Here is a plan of Martin's garden.
The shaded areas show where he plants flowers.

Scale: ☐ = 1 square unit

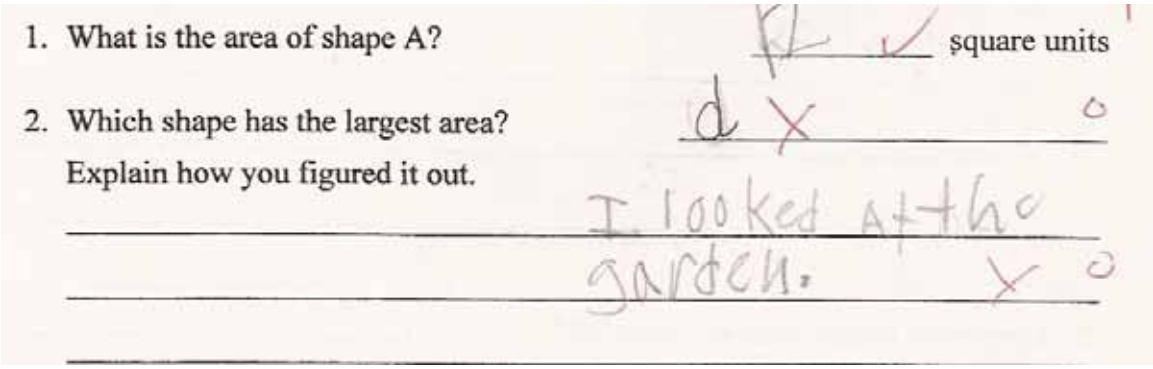
A B C D E

1. What is the area of shape A? 18 X square units

2. Which shape has the largest area? D X has the largest
Explain how you figured it out.
I counted around the distance of the shapes and got their areas

Students G and H both know how to find the area of shape A. However, when trying to compare the areas they try to rely on visual clues giving them an incomplete justification.

Student G

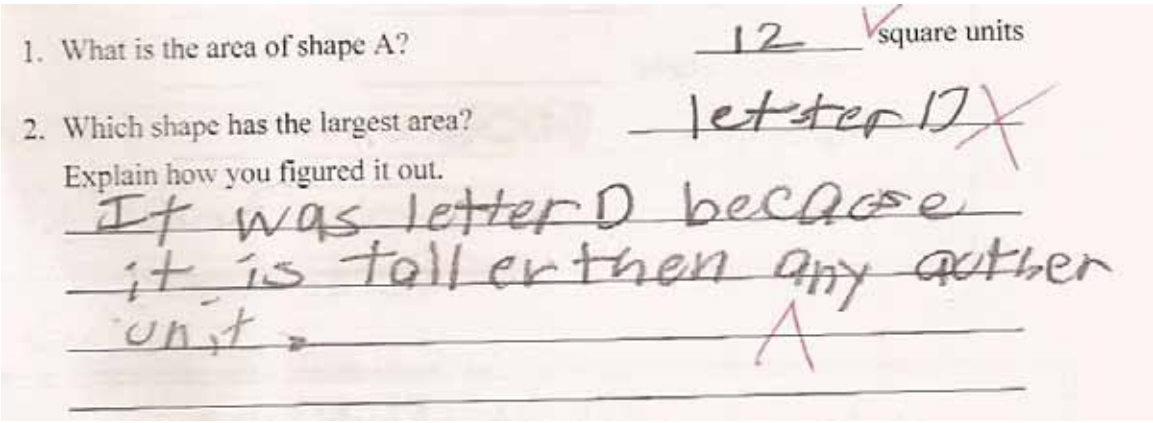

1. What is the area of shape A? square units

2. Which shape has the largest area?

Explain how you figured it out.

I looked at the garden.

Student H


1. What is the area of shape A? 12 square units

2. Which shape has the largest area? letter D

Explain how you figured it out.

It was letter D because it is taller then any other unit.

Third Grade

3rd Grade Task 5 Garden Design

Student Task	Find the area of a rectangle on a grid. Compare area of 4 shapes on a grid, give the values of the area, and pick which is largest. Design a different shape with the same area as the original rectangle.
Core Idea 4 Geometry and Measurement	**Recognize and use characteristics, properties, and relationships of two-dimensional geometric shapes and apply appropriate techniques to determine measurements**. • Calculate perimeter and area and be able to distinguish between the two measures.

Mathematics in the Task

- Ability to find the area of different, irregular shapes/ figures on a grid.
- Ability to make a comparison by first evaluating all the options and then choosing the largest.
- Ability to draw a shape to meet the constraints of a different shape from A and with an area the same as A.

Based on teacher observation, this is what third graders knew and were able to do:

- Find the area of Shape A
- Identify the shape with the largest area
- Draw a shape with the same area as A

Areas of difficulty for third graders:

- Justifying why shape A had the largest area
- Confusing area and perimeter
- Drawing a shape <u>different</u> from A, either not recognizing a rotation or making a shape the same as A.

Strategies used by successful students:

- Labeling the areas of all the shapes

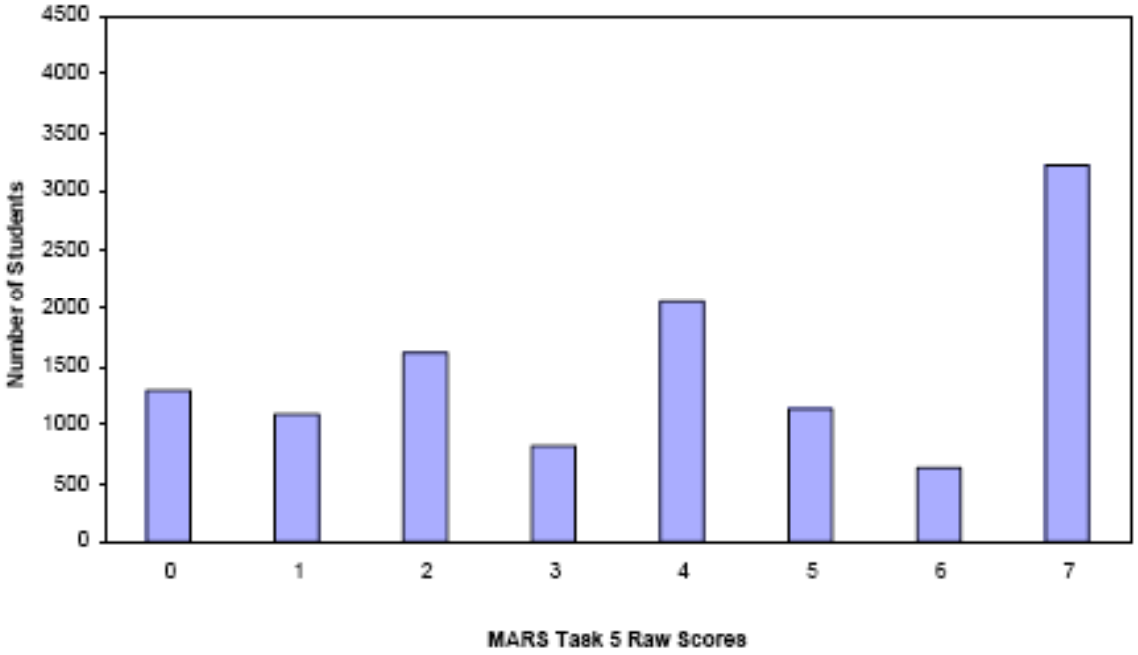
Task 5 – Garden Design

Mean: 3.96 StdDev: 2.44

Table 19: Frequency Distribution of MARS Test Task 5, Grade 3

Task 5 Scores	Student Count	% at or below	% at or above
0	1294	10.9%	100.0%
1	1088	20.1%	89.1%
2	1619	33.8%	79.9%
3	812	40.6%	66.2%
4	2055	58.0%	59.4%
5	1138	67.6%	42.0%
6	625	72.8%	32.4%
7	3220	100.0%	27.2%

Figure 28: Bar Graph of MARS Test Task 5 Raw Scores, Grade 3



The maximum score available on this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Most students, 89%, could find the area of Shape A. Many students, 80%, could find the area of shape A and identify the shape with the largest area. 66% could also draw a rotation of shape A. More than half the students, 59%, could find the area of A, identify the shape with the largest area, and draw a different shape with the same area as A. 27% of the students could meet all the demands of the task, including finding the values of shapes B, C, and D as part of the process of making a comparison. Almost 11% of the students scored no points on this task. 28% of those students did not attempt the task.

Garden Design

Points	Understandings	Misunderstandings
0	72% of the students with this score attempted the task.	Students confused area and perimeter or gave a shape name instead of quantifying area.
1	Students could give the area for shape A.	Students tried to use visual clues like height of width to compare area, rather than using calculations.
2	Students could give the area for A and choose the shape with the largest area.	Students did not know how to justify a comparison. They may have said, "I counted". But, they did not quantify the areas for B, C, and D.
3	Students could give area for A, choose shape with largest area, and draw a rotation for A.	Students did not realize that the rotation is actually the same shape as A. It is not a different shape.
4	Students could give the area for A, choose a shape with the largest area, and design a shape that is different from A but maintains the same area.	Students did not know how to justify a comparison. They may have said, "I counted". But, they did not quantify the areas for B, C, and D.
6		Students were still making rotations for A rather than designing a different shape.
7	Students could give the area for all the shapes and identify the shape with the largest area based on quantity. Students could design a shape with the constraints: different shape than A and same area as A.	

Implications for Instruction

Students need a variety of experiences with geometric shapes. They need to see shapes in a variety of orientations and be able to recognize which are the same and which are different. Being able to draw or design a shape with a given area or perimeter requires synthesizing a variety of ideas about shapes and is significantly different from finding the area of a shape already drawn. Students need opportunities to make their own shapes to meet a set of criteria. Sometimes those criteria should include things that are not possible, like a triangle with two right angles or that push their thinking, like making all the rectangles with a given area.

Students should also have the opportunity to work with optical illusions, or shapes that appear to be larger or smaller, but are actually the same size. This pushes them to see the need to find the exact area rather than relying on visual clues.

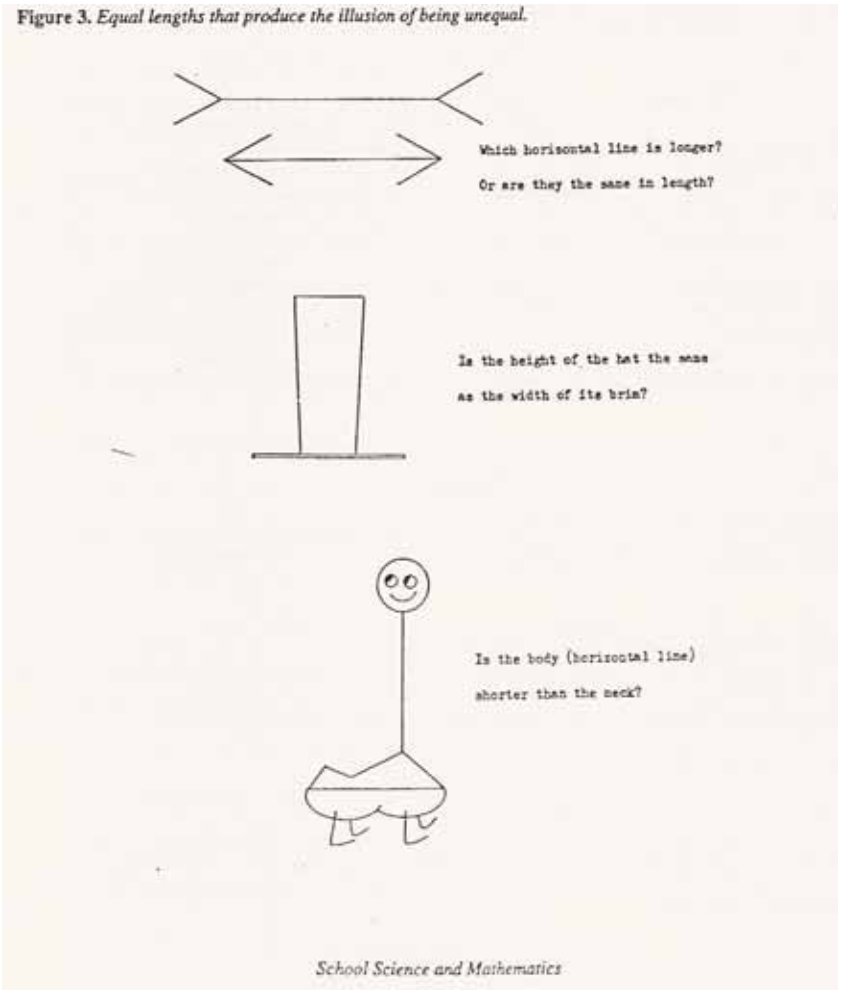
In order to make a comparison of areas or the best deal at the store, it is important to quantify the amounts of all the choices before making a final decision. Students should have the opportunity to look at comparisons in a variety of settings, including money, data, and measurements. Discussion and carefully crafted problems should lead them to understand the value of quantifying each choice before deciding on the rank of the items.

Ideas for Action Research

Visual Clues

Give students some common optical illusions and ask them which is longer or shorter, smaller or larger. Ask them to tell how they knew. After all students have made a commitment to one of the choices, have students measure to find the correct answer. Then discuss why it is important to measure.

Figure 3. *Equal lengths that produce the illusion of being unequal.*



School Science and Mathematics

Now give students some rectangles to look at. Consider the task, Curtis's quilts. The quilts are 1 x 12, 2 x 6, and 3 x 4. Ask them to make an estimate about which is larger and explain their reasoning. Now have them calculate or count the areas of each rectangle.